AUB - <u>Math 218</u> - Spring 2009 **<u>QUIZ 1</u>, March 19** **Duration: 70 min**

Instructor: N. Makhoul-Karam **Sections 1 & 12**

<u>Exercise 1</u>: 15 points

Solve the following linear system by using Gaussian elimination:

$$\begin{cases} x + 2y - 3z = 4 \\ x + 3y + z = 11 \\ 2x + 5y - 4z = 13 \\ 2x + 6y + 2z = 22 \end{cases}$$

<u>Exercise 2</u>: 15 points

For what value(s) of the parameter k has exactly one solution? No solution? Infinitely many solutions?

$$\begin{cases} x \qquad - 3z = -3 \\ x + 2y + kz = 1 \\ 2x + ky - z = -2 \end{cases}$$

<u>Exercise 3</u>: 10 points

Evaluate the determinant of the following matrix:

$$A = \begin{pmatrix} 2 & 5 & -3 & -2 \\ -2 & -3 & 2 & -5 \\ 1 & 3 & -2 & 2 \\ -1 & -6 & 4 & 3 \end{pmatrix}.$$

<u>Exercise 4</u>: 20 points (15 points +5 points)

Let $A = \begin{pmatrix} 1 & 3 & 4 \\ 3 & -1 & 6 \\ -1 & 5 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

a) Justify that the matrix A is invertible and then, find A^{-1} by using a method of your choice.
b) Solve the linear system $Ax = b$ using A^{-1}.

<u>Exercise 5</u>: 15 points

Let $A = \begin{pmatrix} 5 & 2 \\ 9 & 2 \end{pmatrix}$.

a) Find the eigenvalues and the corresponding eigenvectors of the matrix A.
b) Evaluate the trace and the determinant of the matrix A.
   What identity relating $tr(A)$ and the eigenvalues of A can you find?
   What identity relating $det(A)$ and the eigenvalues of A can you find?

<u>Exercise 6</u>: 25 points (5 points for each question)

a) Is it true that any upper triangular matrix is in row-echelon form?
b) Let $A = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$ where $a \in \mathbf{R}$. Find A^2, A^3, A^4 and conjecture a general expression of A^n, for all n.
c) Let $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Find all matrices $\begin{pmatrix} x & y \\ z & w \end{pmatrix}$ that commute with B.
d) Under what condition on n does an $n \times n$ matrix A with real entries satisfy the condition $A^2 = -I$?
e) For any $n \times n$ square matrix A show that: $det(adj(A)) = det(A)^{n-1}$.

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AUB - <u>Math 218</u> - Fall 2008-2009 **<u>QUIZ 1</u>, November 11** **Duration: 70 min**

Instructor: N. Makhoul-Karam **Sections 9**

<u>Exercise 1</u>: (20 points)
Solve the following linear system:

$$\begin{cases} x_2 - x_3 + x_4 = 0 \\ 2x_1 - x_2 + 3x_3 - x_4 = 6 \\ 4x_1 + 4x_2 + 4x_4 = 12 \\ 3x_1 - 3x_2 + 6x_3 - 3x_4 = 9 \end{cases}$$

<u>Exercise 2</u>: (10 points)
Let $L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ a & 1 & 0 & 0 \\ a^2 & a & 1 & 0 \\ a^3 & a^2 & a & 1 \end{pmatrix}$. Find L^{-1}, using row operations.

<u>Exercise 3</u>: (20 points)

Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix}$

a) Find the determinant of *A*, using a cofactor expansion of your choice.
b) Find the determinant of *A*, as a sum of all signed elementary products from *A* (you can use the arrows method).
c) Justify that *A* is invertible and find A^{-1}, using the adjoint formula.

<u>Exercise 4</u>: (15 points)
Let $B = \begin{pmatrix} 2 & 3 \\ 4 & 3 \end{pmatrix}$. Find the eigenvalues and the corresponding eigenvectors of the matrix *B*.

<u>Exercise 5</u>: (10 points)

Consider for any square matrix *A* the following definitions:

If $A^2 = A$, then *A* is called *idempotent*.
If $A^2 = I$, then *A* is called *involutory*.
If $A^n = 0$ for some positive integer *n*, the *A* is called *nilpotent of order n*.

What can you say about the determinant of *A*:

a) If *A* is idempotent?
b) If *A* is involutory?
c) If *A* is nilpotent of order *n*?

<u>Exercise 6</u>: (25 points)

Prove or disprove:

a) $\forall A, B \in R^{n\times n}$, if *BA* = *A*, then *B* = *I* (where *I* is the identity matrix of size *n*).
b) $\forall A \in R^{n\times n}$, if *D* is an *n* x *n* diagonal matrix then *AD* = *DA*.
c) $\forall A, B \in R^{n\times n}, (A+B)(A-B) = A^2 - B^2$.
d) $\forall A, B \in R^{n\times n}$, if *A* is invertible and *B* is not, then *AB* is not invertible.
e) $\forall A \in R^{n\times n}$, $A + A^T$ is symmetric.

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